1880MNRAS..40..482M

and therefore

$$\int ndt = \theta + 3me' \sin n't - \frac{11}{8} m^2 \left(1 - \frac{5}{2} e'^2\right) \sin(2\theta - 2n't)$$
$$- \frac{77}{16} m^2 e' \sin(2\theta - 3n't) + \frac{11}{16} m^2 e' \sin(2\theta - n't)$$
$$+ 3\frac{de'}{ndt} \cos n't + \frac{295}{24} m^2 e' \frac{de'}{ndt} \cos(2\theta - 2n't) - \frac{413}{48} m^2 \frac{de'}{ndt} \cos(2\theta - 3n't)$$
$$+ \frac{59}{48} m^2 \frac{de'}{ndt} \cos(2\theta - n't).$$

Hence θ differs from $\int ndt$ by periodic terms only, which proves the proposition.

The value of $\frac{dn}{ndt}$ above found agrees with that found in my paper published in the 'Philosophical Transactions' for 1853.

Note on the Constant of Lunar Parallax. By Professor J. C. Adams, M.A., F.R.S.

From the report of a discussion which took place at a late meeting of the Society, I have reason to believe that an explanation of the apparent discrepancy between the value of the constant of parallax given by me in the Appendix to the *Nautical Almanac* for 1856, and in the *Monthly Notices*, vol. xiii. p. 263, and the value of the constant found by Hansen in the Introduction to his Lunar Tables, may not be unacceptable to some of our members.

It will be proper to begin this explanation by recalling to mind that my formula, in the article of the *Monthly Notices* above referred to, does not represent the parallax itself, but rather the sine of that quantity converted into seconds of arc by dividing by sin $1''$ or, which is the same thing, by multiplying by the number of seconds in the arc equal to the radius. The employment of the sine of the parallax instead of the parallax itself appears to be desirable both on theoretical as well as practical grounds.

In the first place, the sine of the parallax, being proportional to the reciprocal of the radius vector, is the quantity given directly by the lunar theory, and, in the next place, it is the same quantity which is wanted in the reduction of lunar observations.

What I have called the constant of parallax in the papers above referred to is, then, the constant term in the expression for the converted sine of the parallax, supposing the periodic

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terms to be expressed in cosines of angles which increase in proportion to the time. The value found for this constant was $3422''\cdot325$.

This quantity may also be called very appropriately the mean sine of the parallax, although I do not use the term in the papers referred to.

The value of the corresponding constant in the expression of the parallax itself is $0''\cdot157$ greater than this, or $3422''\cdot48$, which may appropriately be called the mean parallax.

The formula in the Introduction to Hansen's Lunar Tables does not give the sine of the parallax, but the *logarithm* of the sine of the parallax, and the constant which Hansen calls C is a quantity such that the constant term in his expression for the logarithm of sine of the parallax is

$$\log \sin C.$$

Now, it is plain that the constant term in the development of log sin parallax is a different quantity from the logarithm of the constant term of the sine of the parallax, and hence my constant of parallax differs from Hansen's quantity

$$\frac{\sin C}{\sin 1''}.$$

We may readily express the relation between these two constants in the case in which the orbit is supposed to be an undisturbed ellipse.

In this case, if the reciprocal of the radius vector, which is proportional to the sine of the parallax, be developed in terms of cosines of multiples of the mean anomaly,

then, a being the semi-axis major,
and e the eccentricity of the orbit,

the constant term in the development will be $\frac{1}{a}$.

In the same case, the constant term in the development of the logarithm of the reciprocal of the radius vector, expressed in terms of the same form as before, will be

$$\log \frac{1}{a}\left(1-\frac{1}{4}e^2\right)$$

very nearly, instead of $\log \frac{1}{a}$; so that if c denote the constant term in the former development, and $\log c'$ the constant term in the latter, we shall have

$$\frac{c'}{c} = 1-\frac{1}{4}e^2 \text{ very nearly.}$$

This relation will still be approximately though not exactly satisfied when the Moon's perturbations are taken into account.

Hansen himself, in a paper in the 17th volume of the *Astronomische Nachrichten*, p. 299, in which he gives the results which

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1880MNRAS..40..482M

he had obtained in a preliminary investigation of the lunar perturbations, finds that the number corresponding to the constant term in the logarithm of the sine of the parallax requires to be augmented by $2''\cdot71$ in order to reduce it to the constant term in the sine of the parallax itself.

Calling the parallax p, Hansen finds that the value of the constant term in $\log\left(\frac{\sin p}{\sin 1''}\right)$ is

$$\log (3419''\cdot35),$$

and hence he concludes that the constant term in $\left(\frac{\sin p}{\sin 1''}\right)$ is

$$3422''\cdot06.$$

By repeating Hansen's calculation and taking into account some small terms omitted by him, I find the amount of the reduction to be slightly less than the above, viz. $2''\cdot67$, so that the constant term in

$$\frac{\sin p}{\sin 1''}$$

according to Hansen's preliminary theory would be $3422''\cdot02$.

This value, however, is not immediately comparable with my own, being founded on different elements.

Both values are purely theoretical, depending on the ratio of the Moon's mass to that of the Earth, the ratio of the Earth's equatorial and polar axes, and the ratio of the Earth's radius to the length of the seconds' pendulum in a given latitude.

If M denote the mass of the Earth,

m that of the Moon,

A the Earth's equatorial radius,

R the Earth's radius at a point of which the sine of the latitude is $\frac{1}{\sqrt{3}}$,

P the length of the seconds' pendulum at the same point;

then the constant term of the sine of the horizontal parallax corresponding to the latitude just specified may be represented by

$$\left(\frac{M}{M+m}\cdot\frac{R}{P}\right)^{\frac{1}{3}}F,$$

and therefore the constant term of the sine of the equatorial horizontal parallax may be represented by

$$\frac{A}{R}\left(\frac{M}{M+m}\cdot\frac{R}{P}\right)^{\frac{1}{3}}F = \left(\frac{M}{M+m}\cdot\frac{A^3}{R^2P}\right)^{\frac{1}{3}}F,$$

where F is a factor which may be found by theory from elements which may be considered as known with all desirable accuracy.

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1880MNRAS..40..482M

The values of $\frac{M}{m}$, A, R and P employed in finding my constant are the following:—

$$\frac{M}{m} = 81{\cdot}5,$$

which corresponds very nearly to Dr. Peters' constant of Nutation;

$$A = 20923505 \text{ English feet,}$$
$$R = 20900320 \quad ,,$$
$$P = 3{\cdot}256989 \quad ,,$$

R and P belong to a point the sine of the *geographical* latitude of which is $\frac{1}{\sqrt{3}}$.

A and R are the quantities found from Bessel's latest determination of the figure and dimensions of the Earth as given in *Astron. Nachr.* vol. xix., p. 216, supposing that

$$1 \text{ Toise} = 6{\cdot}394564 \text{ English feet.}$$

P is found thus: according to the formula given in p. 94 of Baily's Report on Foster's Pendulum experiments, *Mem. of the R.A.S.* vol. vii., the square of the number of vibrations made in a mean solar day, at a point the sine of whose geographical latitude is $\frac{1}{\sqrt{3}}$, by a pendulum which vibrates seconds in London is

$$7441625711 + \frac{1}{3}(38286335) = 7454387823.$$

Also Captain Kater's determination of the length of the seconds' pendulum in London is

$$39{\cdot}13929 \text{ inches} = 3{\cdot}2616075 \text{ feet.}$$

Hence as the square of the number of vibrations made at a given place in a given time varies inversely as the length of the pendulum, we derive the value above given for P.

The values of the fundamental elements employed by Hansen are the following:—

$$\frac{M}{m} = 80$$
$$A = 6377157 \quad \text{metres}$$
$$R_1 = 6370063 \quad ,,$$
$$P_1 = 0{\cdot}992666 \quad ,,$$

and R_1 and P_1 belong to a point the sine of the *geocentric* latitude of which is $\frac{1}{\sqrt{3}}$.

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1880MNRAS..40..482M

The corresponding values of R and P for a point the sine of whose geographical latitude is $\frac{1}{\sqrt{3}}$ are the following:—

$$R = 6370126 \quad \text{metres}$$
$$P = 0{\cdot}992651 \quad ,,$$

And the constant term of the sine of the equatorial horizontal parallax may be represented either by

$$\left(\frac{M}{M+m}\;\frac{A^3}{R^2P}\right)^{\frac{1}{3}}F, \quad \text{or by} \quad \left(\frac{M}{M+m}\;\frac{A^3}{R_1{}^2P_1}\right)^{\frac{1}{3}}F_1.$$

In my calculation of the factor F, I took into account terms of the order of the square of the Earth's compression. It would otherwise have been useless to distinguish between R^2P and $R^2{}_1P_1$ or between F and F_1.

At the time when Hansen's paper appeared in the *Astron. Nachr.* Bessel's latest determination of the figure and dimensions of the Earth was not available. Hansen employed an earlier determination given by Bessel in *Astron. Nachr.* vol. xiv., p. 344, in which the results were affected by an error in the calculation of the French arc of the meridian which was discovered later.

Hence the corrections to be applied to the logarithms employed by Hansen in order to make them agree with those employed by me are the following, expressed in units of the 7th decimal:—

	Correction.
$\log\left(\frac{M}{M+m}\right)$	+987
$\log\left(\frac{A}{R}\right)$	+ 25
$\log\left(\frac{R}{P}\right)$	−150

The correction to be applied to Hansen's value of the logarithm of the constant term in the sine of the parallax is therefore

$$25+\frac{1}{3}(987-150) = 304 \quad \text{of the same units.}$$

And the corresponding correction of the constant term of the sine of the parallax will be $0''{\cdot}24$, and therefore according to Hansen's preliminary theory, employing my system of fundamental data, the value of this constant term will be $3422''{\cdot}26$.

In my independent transformation of Hansen's expression I found the rather more precise value $3422''{\cdot}264$.

This is less than my own value of the same constant by $0''{\cdot}06$ nearly, as stated in my paper in the Appendix to the *Nautical Almanac* for 1856.

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1880MNRAS..40..482M

I there intimated my belief that Hansen's definitive theory would probably be found to introduce a correction to his former value of the constant term in question, and this turns out to be the case.

In *Astron. Nachr.* vol. xvii., p. 298, the constant term in $-w$ which denotes the perturbations of the natural logarithm of the reciprocal of the radius vector, divided by sin 1″, is given as 1345″·281, but in the introduction to Hansen's Lunar Tables this same quantity is given as 1348″·840. Hence, the correction to the former value is 3″·559, and multiplying this by sin 1″ and by 3422″ we find the corresponding correction of the constant of parallax to be 0″·059, so that this constant becomes 3422″·323, a result which agrees perfectly with my own.

In this connection it may be worth mentioning that the only periodic term in which I found any difference much exceeding 0″·01 between my coefficients of parallax and those obtained by a transformation of the results of Hansen's preliminary theory was that which has the argument denoted by $t+z$ in Damoiseau's notation.

The corresponding term in $-w$ is in Hansen's preliminary theory

$$10''\!\cdot\!92 \cos (t+z),$$

whereas in the Introduction to the Lunar Tables this term is

$$8''\!\cdot\!73 \cos (t+z);$$

the correction to the coefficient is $-2''\!\cdot\!19$, and multiplying this as before by sin 1″ and by 3422″ we find the correction to the corresponding term of the sine of the parallax to be

$$-0''\!\cdot\!036 \cos (t+z),$$

and if this be applied to the value of this term in the preliminary theory, viz.

$$0''\!\cdot\!181 \cos (t+z),$$

the result is

$$0''\!\cdot\!145 \cos (t+z),$$

which agrees perfectly with my own.

It should be remarked that, in the Introduction to his Lunar Tables, Hansen still continues to use the same fundamental data as he had done in his earlier paper, so that the value of the constant term in the sine of the parallax according to the data adopted in the Tables is 3422″·08.

Note added June 17, 1880.

In Professor Newcomb's valuable transformation of Hansen's Lunar Theory which I have just received, it is wrongly assumed

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1880MNRAS..40..488M

that I employed the same data as Hansen for the figure and dimensions of the Earth, and that my value of P, viz. 3·256989 feet, relates like Hansen's to a point the sine of whose *geocentric* latitude is $\frac{1}{\sqrt{3}}$, whereas it should be the *geographical* latitude, as that is the latitude which enters into Baily's formula from which my value of P is deduced.

In consequence of this, Professor Newcomb finds a discrepancy of $0''\cdot03$ between Hansen's value of the constant of parallax and mine when both are derived from the same system of fundamental data; but it has been shown above that no such discrepancy exists.

By a typographical error, the value of P which Professor Newcomb quotes from me is printed as 3·256 89 feet, instead of 3·256989 feet.

Ephemeris for finding the Positions of the Satellite of Neptune, 1880–81. By A. Marth, Esq.

P, angle of position of the minor axis of the apparent orbit in the direction of superior conjunction.

a, *b*, major and minor semi-axis of the apparent orbit.

Long. = longitude of the satellite in its orbit, reckoned from the point which is in superior conjunction with the planet.

Gr. Noon.	P.	*a*.	*b*.	Log. *a*.	Log. *b*.	Long.	Diff.
1880.	°	″	″			°	°
Sept. 1	314·30	16·66	6·51	1·2216	0·8136	112·48	612·51
11	314·24	16·74	6·53	·2238	·8150	4·99	·46
21	314·14	16·81	6·54	·2257	·8156	257·45	·41
Oct. 1	314·01	16·88	6·54	1·2273	0·8156	149·86	·37
11	313·87	16·93	6·53	·2285	·8149	42·23	·34
21	313·71	16·96	6·51	·2294	·8135	294·57	·32
31	313·55	16·97	6·48	1·2298	0·8115	186·89	612·30
Nov. 10	313·38	16·97	6·44	·2297	·8089	79·19	·30
20	313·22	16·95	6·40	·2292	·8059	331·49	·31
30	313·07	16·91	6·35	1·2282	0·8025	223·80	·32
Dec. 10	312·93	16·86	6·29	·2268	·7989	116·12	·35
20	312·82	16·79	6·24	·2251	·7952	8·47	·38
30	312·74	16·71	6·19	1·2231	0·7917	260·85	612·43
1881.							
Jan. 9	312·68	16·63	6·14	·2208	·7884	153·28	·48
19	312·66	16·53	6·10	·2183	·7854	45·76	·53
29	312·67	16·44	6·06	1·2158	0·7828	298·29	·58
Febr. 8	312·72	16·34	6·04	·2133	·7808	190·87	·64
18	312·79	16·25	6·02	·2109	·7795	83·51	612·69
28	312·90	16·17	6·01	1·2088	0·7790	336·20	

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